

**Converting Binary (base 2) to Denary (base 10)**

Say we are converting the binary number 1101 0110 into denary; add the following headings over each number:

128	64	32	16	8	4	2	1
1	1	0	1	0	1	1	0

Then simply multiply to find the total:

$$(1 \times 128) + (1 \times 64) + (0 \times 32) + (1 \times 16) + \dots = \underline{214}$$

**Converting Denary (base 10) to Binary (base 2)**

The reverse process is to take out the largest number (power of 2) you can, like this:

214 – can we take out 128?	Yes	1	Remainder = 86
86 – can we take out 64?	Yes	1	Remainder = 22
22 – can we take out 32?	No	0	Remainder = 22
22 – can we take out 16?	Yes	1	Remainder = 6
6 – can we take out 8?	No	0	Remainder = 6
6 – can we take out 4?	Yes	1	Remainder = 2
2 – can we take out 2?	Yes	1	Remainder = 0
0 – can we take out 1?	No	0	Remainder = 0

Answer = 1101 0110

**Converting Binary (base 2) to Hexadecimal (base 16)**

Say we are converting the binary number 1101 0110 into hexadecimal; split the number into two 4-bit nibbles and convert them into denary (*if the number does not have the right number of digits, simply add zeros to the LHS*).

$$1101 = 13 \qquad 0110 = 6$$

Then, convert each denary number into a single hex digit (where 10 = A, 11 = B, 12 = C, 13 = D, 14 = E, 15 = F)

$$13 = D \qquad 6 = 6$$

Therefore:

$$\underline{1101\ 0110 = D6}$$

**Converting Hexadecimal (base 16) into Binary (base 2)**

Lets convert D6 back into binary. First convert each hex digit in to a denary number and then convert that into binary:

$$D6 = 13\ 6$$

$$13 = (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1) = 1101$$

$$6 = (0 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1) = 0110$$

Therefore:

$$\underline{D6 = 1101\ 0110}$$

**Adding binary numbers**

Lets add 0110 1010 to 00101101

Note: = 106 + 45 = 151

First, write them out like this:

```
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
```

Just like denary adding, add the two digits together and carry the 1 if necessary:

```
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      1
```

```
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      1 1
```

```
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      1 1 1
```

```
      1
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      0 1 1 1
```

```
      1
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      1 0 1 1 1
```

```
      1 1
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      0 1 0 1 1 1
```

```
      1 1 1
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      0 0 1 0 1 1 1
```

```
      1 1 1
0 1 1 0 1 0 1 0
0 0 1 0 1 1 0 1 +
      1 0 0 1 0 1 1 1
```

Note: = 151

**Negative binary numbers – 2s complement**

With 8 bits we can store any **positive integer** from 0 to 255. But what about **negative integers**?

The answer is to change the range from -128 to +127, using the first bit to indicate the sign; thus **1000 000** would indicate that the number is **negative** and the lowest possible number (-128) and **0111 1111** would be both **positive** and the highest possible number (+127).

This means that all the positive numbers still work as expected (simply ignoring the leading 0) and you don't end up with two 0 value (positive 0 and negative 0).

**To convert a negative denary number into binary using 2s complement**

- Convert the positive number into binary
- Invert each bit
- Add 1

e.g. -127

Convert to binary: 0111 1111  
 Invert each bit: 1000 0000  
 Add 1: 1000 0001

e.g. -37

Convert to binary: 0010 0101  
 Invert each bit: 1101 1010  
 Add 1: 1101 1011

**To convert a negative binary number into denary using 2s complement**

Simply reverse the process:

e.g. 1000 0001

Subtract 1: 1000 0000  
 Invert each bit: 0111 1111  
 Convert to denary: -127

e.g. 1101 1011

Subtract 1: 1101 1010  
 Invert each bit: 0010 0101  
 Convert to denary: -37

**Subtracting binary numbers**

Rather than subtracting a positive number, try adding a negative number.

$$+106 - +45 = +106 + -45$$

Lets work out 0110 1010 - 00101101

$$\text{Note: } = 106 - 45 = 61$$

First, use the 2s complement to invert the second number:

$$-0010 1101 = +(1101 0010 + 1) = 1101 0011$$

Then do the addition:

$$\begin{array}{r} 01101010 \\ \underline{11010011} + \\ 100111101 \end{array}$$

Discard any leading digits (remember the first digit just indicates the sign - +/-)

$$0011 1101 = 61$$